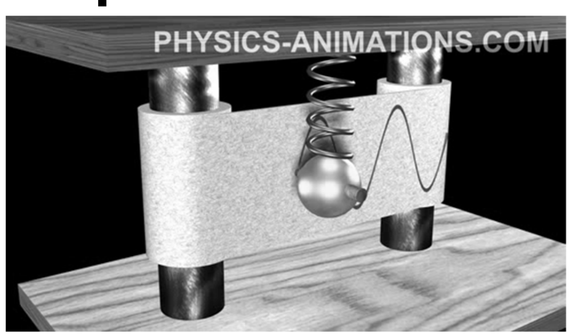
Simple Harmonic Motion

Springs - Part 2

1

Displacement in SHM

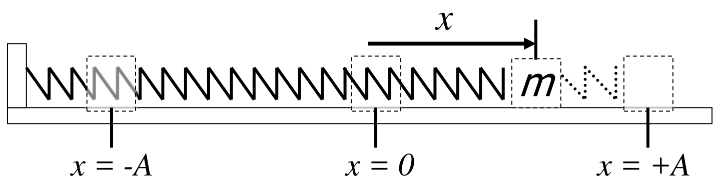


- As seen in our lab, when a mass hanging from a spring is pulled away from its equilibrium position, it will be put into simple harmonic motion.

2

Displacement in SHM

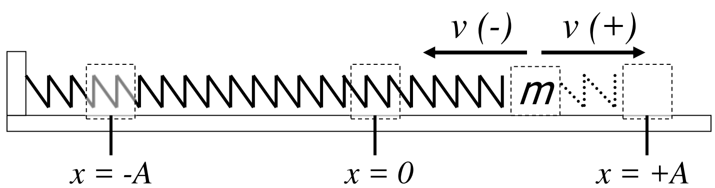
- The same motion can be observed in a mass attached to a horizontal spring oscillating on a frictionless surface.



- Displacement is positive when the position is to the right of the equilibrium position (x = 0) and negative when located to the left.
- The maximum displacement is called the amplitude, A.

3

Velocity in SHM



- Velocity is positive when moving to the right and negative when moving to the left.
- It is zero at the end points and a maximum at the midpoint in either direction (+ or -).

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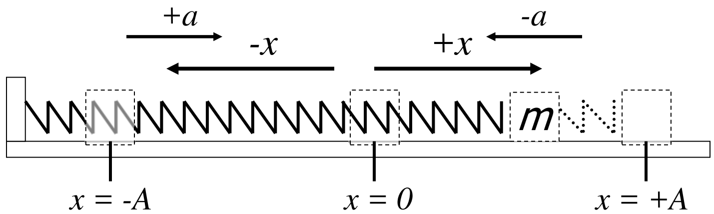
Acceleration in SHM



- Acceleration is in the direction of the restoring force. (*a* is positive when x is negative, and negative when x is positive.)

$$F = ma = -kx$$

- Acceleration is a maximum at the end points and it is zero at the center of oscillation.

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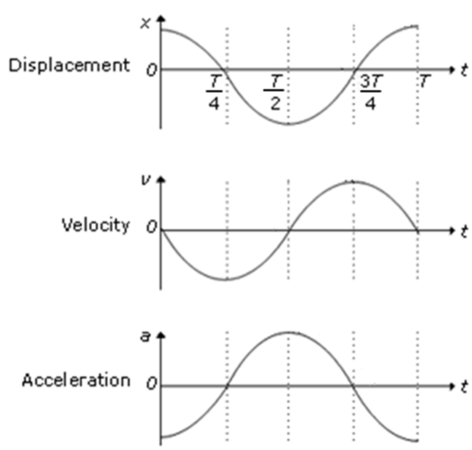
Comparing Displacement, Velocity and Acceleration of SHM



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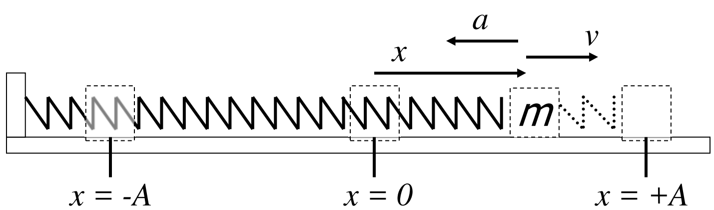
Acceleration vs. Displacement



Given the spring constant, the displacement, and the mass, the acceleration can be found from:

$$F = ma = -kx \quad \text{or} \quad a = \frac{-kx}{m}$$

Note: Acceleration is always opposite to displacement.

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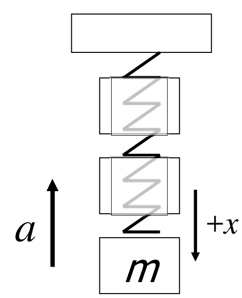
Example 1: A 2-kg mass hangs at the end of a spring whose constant is k = 400 N/m. The mass is displaced a distance of 12 cm and released. What is the acceleration at the instant the displacement is x = +7 cm?



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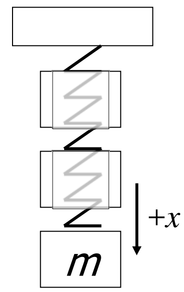
Example 2: What is the maximum acceleration for the 2-kg mass in the previous problem? (A = 12 cm, k = 400 N/m)

The maximum acceleration occurs when the restoring force is a maximum; i.e., when the stretch or compression of the spring is largest.

$$F = ma = -kx \qquad x_{max} = \pm A$$



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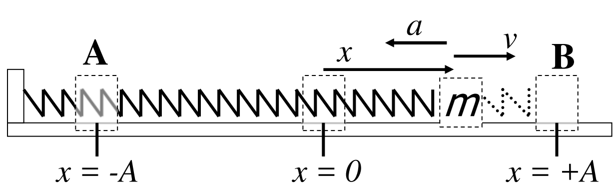
Energy of a Vibrating System:



- At points A and B, the velocity is zero and the acceleration is a maximum. The total energy is:

$$U + K = \tfrac{1}{2}kA^2 \quad x = \pm A \text{ and } v = 0.$$

- At any other point: $U + K = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2$

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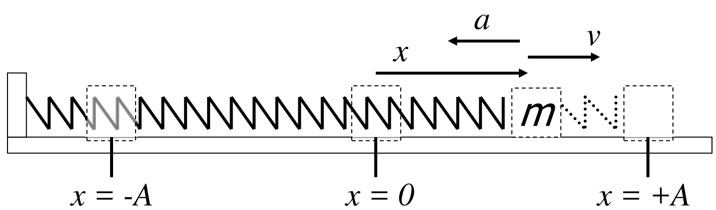
Conservation of Energy

The total mechanical energy (U + K) of a vibrating system is constant; i.e., it is the same at any point in the oscillating path.



For any two points A and B, we may write:

$$K_A + U_A = K_B + U_B$$

$$\tfrac{1}{2}mv_A^2 + \tfrac{1}{2}kx_A^{\,2} = \tfrac{1}{2}mv_B^2 + \tfrac{1}{2}kx_B^{\,2}$$

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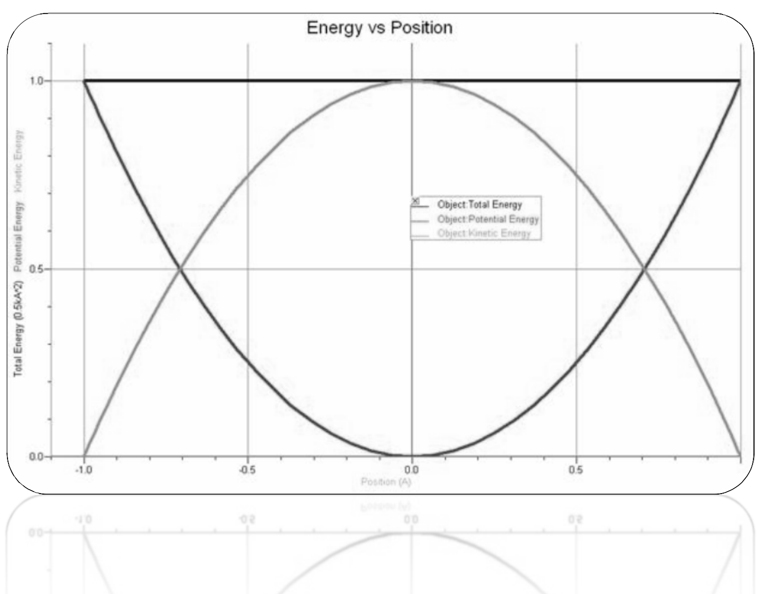
Energy in SHM



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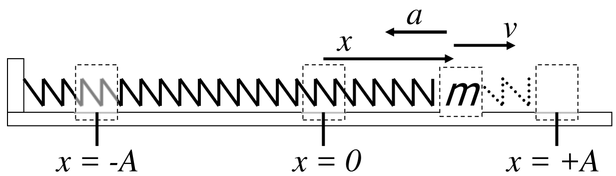
Velocity as Function of Position



$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2 \qquad v = \sqrt{\frac{k}{m}}\sqrt{A^2 - x^2}$$

v_{max} when x = 0: $$v_{\max} = \sqrt{\frac{k}{m}}A$$

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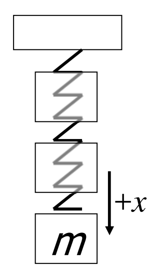
Example 3: A 2-kg mass hangs at the end of a spring whose constant is k = 800 N/m. The mass is displaced a distance of 10 cm and released. What is the velocity at the instant the displacement is x = +6 cm?



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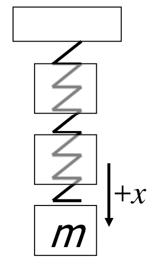
<u>Example 3 (Cont.):</u> What is the maximum velocity for the previous problem?
(A = 10 cm, k = 800 N/m, m = 2 kg.)

The velocity is maximum when x = 0:



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The Period and Frequency as a Function of a and x.

- From our lab we found that the quadratic relationship between period (T) and mass (m)

$$T = 2\pi\sqrt{\frac{m}{k}}$$

- From Newton's 2nd Law and Hooke's Law we found:

$$a = \frac{-kx}{m} \Rightarrow \frac{a}{x} = -\frac{k}{m} \Rightarrow \frac{m}{k} = -\frac{x}{a}$$

Substituting we find…

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The Period and Frequency as a Function of *a* and *x*.

For any body undergoing simple harmonic motion:

$$T = 2\pi\sqrt{\frac{-x}{a}}$$

And since $f = \frac{1}{T}$ $\Rightarrow$ $f = \frac{1}{2\pi}\sqrt{\frac{-a}{x}}$

The frequency and the period can be found if the displacement and acceleration are known. Note that the signs of a and x will always be opposite.

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Acceleration as a Function of *f* and *x*.

- Rewriting the previous equation we get:

$$f = \frac{1}{2\pi}\sqrt{\frac{-a}{x}} \Rightarrow -a = 4\pi^2 f^2 x$$

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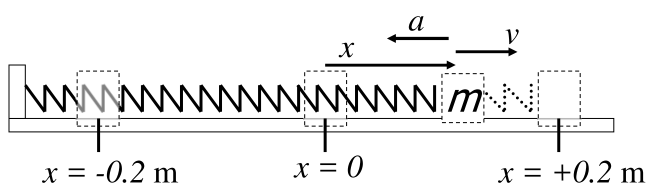
Example 4: The frictionless system shown below has a 2-kg mass attached to a spring (k = 400 N/m). The mass is displaced a distance of 20 cm to the right and released. What is the frequency of the motion?



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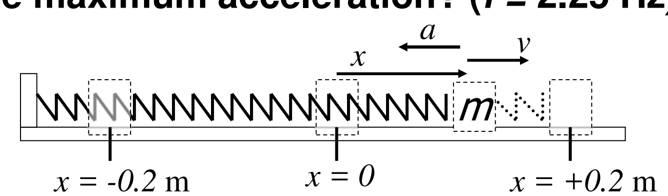
Example 4 (Cont.): Suppose the 2-kg mass of the previous problem is displaced 20 cm and released (k = 400 N/m). What is the maximum acceleration? (*f* = 2.25 Hz*)*



Acceleration is a maximum when $x = \pm A$

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