# Simple Harmonic Motion

Springs - Part 1

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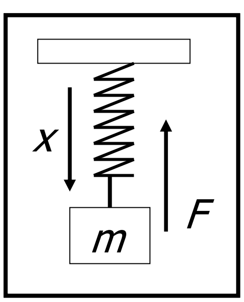
## Hooke's Law

*When a spring is stretched, there is a restoring force that is proportional to the displacement.*




$F = -kx$

*The spring constant k is a property of the spring given by:*

$$k = \frac{\Delta F}{\Delta x}$$

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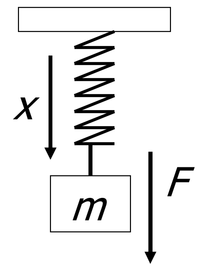
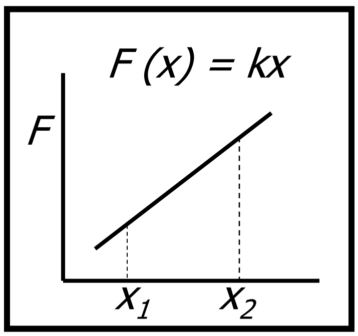
## Work Done in Stretching a Spring

*Work done ON the spring is positive; work BY spring is negative.*




*From Hooke's law the force F is:*




*To stretch spring from $x_1$ to $x_2$, work is:*

$$Work = \tfrac{1}{2}kx_2^2 - \tfrac{1}{2}kx_1^2$$

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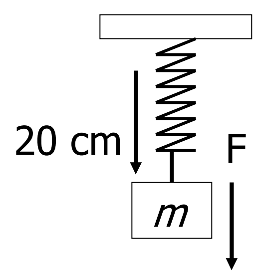
**<u>Example 1:</u> A 4-kg mass suspended from a spring produces a displacement of 20 cm. What is the spring constant?**




*The stretching force is the weight (W = mg) of the 4-kg mass:*

$F = (4 \text{ kg})(9.8 \text{ m/s}^2) = 39.2 \text{ N}$

*Now, from Hooke's law, the force constant k of the spring is:*

$$k = \frac{\Delta F}{\Delta x} = \frac{39.2 \text{ N}}{0.2 \text{ m}}$$

$k = 196$ N/m

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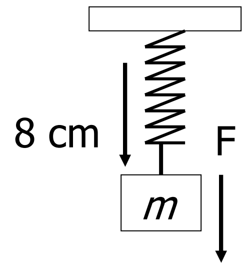
**<u>Example 1(cont.):</u> The mass *m* is now stretched a distance of 8 cm and held. What is the potential energy? (k = 196 N/m)**

The potential energy is equal to the work done in stretching the spring:




$$Work = \tfrac{1}{2}kx_2^2 - \tfrac{1}{2}kx_1^2 \quad (\tfrac{1}{2}kx_1^2 \to 0)$$

$$U = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(196\text{ N/m})(0.08\text{m})^2$$

$$U = 0.627\text{ J}$$

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## Period and Frequency as a Function of Mass and Spring Constant.

For a vibrating body with an elastic restoring force:

Recall that $F = ma = -kx$:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

The frequency *f* and the period *T* can be found if the spring constant *k* and mass *m* of the vibrating body are known. Use consistent SI units.

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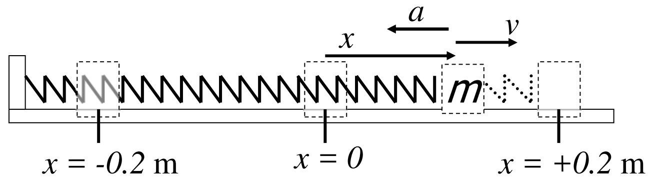
**Example 2:** The frictionless system shown below has a 2-kg mass attached to a spring (k = 400 N/m). The mass is displaced a distance of 20 cm to the right and released. What is the frequency of the motion?




$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{400\text{ N/m}}{2\text{ kg}}}$$

$$\boxed{f = 2.25\text{ Hz}}$$

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