Rotational Momentum and Energy

Rotational Momentum

- The strength of an objects rotational motion.
- For a rigid body:

$$L = I\omega$$

- For a particle moving in a circle:

$$L = \mathrm{m}r^2\left(\frac{v}{r}\right)$$
$$L = mvr$$

Translational Equation:
$p = mv$

Why Does This Happen?

- What happens to ω?
  INCREASES
- What happens to the shape of the skater? RADIUS DECREASES
- How does that change I?
  DECREASES
- Does the angular momentum change? No. $L = I\omega$
  $\downarrow \uparrow \Rightarrow -$

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Changing Rotational Momentum

- Similar to the how applying a force over time to a object moving a straight line will change its linear momentum ($\Delta p=F\Delta t$),
- Applying a torque to a rotating object for a certain amount of time will change its rotational momentum:

$$\Delta L = \tau \Delta t$$

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Conservation of Rotational Momentum

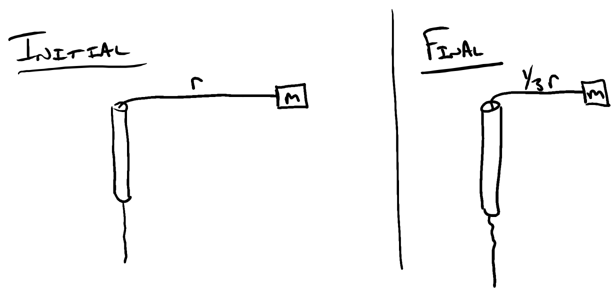
- Again, similar to linear momentum always being conserved in an isolated system, angular momentum is also conserved in an isolated system.

$$L_{initial} = L_{final}$$

No Collision Sample Problem 1

- During our centripetal force lab, a student notices that if the string is pulled through the tube while the stopper is rotating its velocity increases. If the string is pulled so the radius is one third of its original length, what would its new velocity be?

No Collision Sample Problem 1



$$L_{INITIAL} = L_{FINAL}$$

$$I_0\omega_0 = I\omega$$

$$\cancel{m}r^2\omega_0 = \cancel{m}\left(\tfrac{1}{3}r\right)^2\omega$$

$$\cancel{r^2}\omega_0 = \tfrac{1}{9}\cancel{r^2}\omega$$

$$9\omega_0 = \omega$$

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No Collision Sample Problem 2

- A man stands at the center of a circular platform holding his arms extended horizontally with a 4.0 kg block in each hand. The platform is rotating freely at 0.5 rev/s. The moment of inertia of the platform and man is 1.6 kg•m^2. The blocks are 90.0 cm from the axis of rotation. The man pulls his arms in toward his body until they are only 15.0 cm from the axis of rotation. Find his new angular velocity.

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No Collision Sample Problem 2

$$L_0 = L$$
$$I_0 \omega_0 = L\omega$$
$$(I_{M+P} + I_{0_{BLOCK_1}} + I_{0_{BLOCK_2}})\omega_0 = (I_{M+P} + I_{BLOCK_1} + I_{BLOCK_2})\omega$$
$$(1.6 + m_1 r_{0_1}^2 + m_2 r_{0_2}^2)\omega_0 = (1.6 + m_1 r_1^2 + m_2 r_2^2)\omega$$
$$[1.6 + (4)(.9)^2 + (4)(.9)^2]\omega_0 = [1.6 + 4(.15)^2 + 4(.15)^2]\omega$$
$$8.08\omega_0 = 1.78\omega$$
$$\omega_0 = .5\,\frac{REV}{S} \times 2\pi\,RAD = 3.14\,\frac{RAD}{S}$$
$$(8.08)(3.14) = 1.78\omega$$
$$14.3\,\tfrac{RAD}{S} = \omega$$

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Collision Problem

- A disk is rotating freely at 1800 rev/min about a vertical axis through its center. A second disk mounted on the same shaft above the first is initially at rest. The moment of inertia of the second disk is twice that of the first. The second disk is dropped onto the first one, and the two eventually rotate together with a common angular velocity. Find the new angular velocity.

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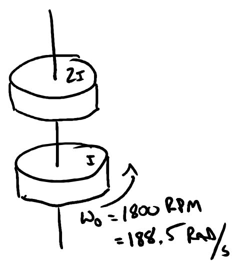
Collision Problem



$I_0 \omega_0 = I \omega$

$I(188.5) = (I + 2I)\omega$

$188.5(\cancel{I}) = 3\cancel{I}\omega$

$\underline{62.3 \text{ RAD/s}} = \omega$

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Rotational Kinetic Energy

- The energy of a object has because of its rotational motion.

$$K_{Rot.} = \frac{1}{2} I \omega^2$$

- The net work done on an object is equal to the change in its energy

$$W = K_{final} - K_{initial}$$

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Rotational Kinetic Energy

- A 15.0 g coin with a diameter of 1.5 cm is spinning about its vertical diameter at 10.0 rev/s. Find its kinetic energy. (I =¼mr²)

$\Downarrow$

62.8 RAD/s

$K_{ROT} = \frac{1}{2} I \omega^2$

$= \frac{1}{2}\left(\frac{1}{4} m r^2\right)\omega^2$

$= \frac{1}{8}(.015\,kg)(.0075)^2(62.8)^2$

$= \underline{4.16 \times 10^{-4}\ J}$

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Total Kinetic Energy

- A rolling object has both translational and rotational energy.
- The total kinetic energy can be found by adding them together.

$$K_{Total} = K_{Translational} + K_{Rotational}$$

$$K_{Total} = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

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Total Kinetic Energy Problem

- A uniform, solid 1.0 kg cylinder rolls at a speed of 1.8 m/s on a flat surface.
  - What is the total kinetic energy of the cylinder?
  - What is the percentage of the total kinetic energy is rotational?
  - Redo the problem with a sphere.

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Total Kinetic Energy Problem

$$K_T = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$
$$= \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{1}{2}m\cancel{r^2}\right)\left(\tfrac{v}{\cancel{r}}\right)^2$$
$$= \tfrac{1}{2}mv^2 + \tfrac{1}{4}mv^2$$
$$= \tfrac{3}{4}mv^2 = \tfrac{3}{4}(1\,\text{kg})(1.8)^2$$
$$= \underline{2.43\,\text{J}}$$
$$\%K_{ROT} = \frac{K_{ROT}}{K_T} = \frac{\tfrac{1}{4}mv^2}{\tfrac{3}{4}mv^2}$$
$$= \tfrac{1}{3} \Rightarrow \underline{33\%}$$

$$K_T = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$
$$= \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{2}{5}mr^2\right)\left(\tfrac{v}{\cancel{r}}\right)^2$$
$$= \tfrac{1}{2}mv^2 + \tfrac{1}{5}mv^2$$
$$= \tfrac{7}{10}mv^2 = \tfrac{7}{10}(1\,\text{kg})(1.8)^2$$
$$= \underline{2.27\,\text{J}}$$
$$\%K_{ROT} = \frac{K_{ROT}}{K_T} = \frac{\tfrac{1}{5}mv^2}{\tfrac{7}{10}mv^2}$$
$$= \tfrac{2}{7} = \underline{28.5\%}$$

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Conservation of Energy

- In a closed and isolated system, the total amount of energy must remain constant.
- The energy can change from one form to another.

$$(U + K_T + K_R)_{initial} = (U + K_T + K_R)_{final}$$

$$(mgh + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2)_{initial} = (mgh + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2)_{final}$$

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Conservation of Energy Problem 1

- A cylinder is rolling at a rate of 1.26 m/s on a flat surface before it reaches the base of a 30⁰ incline. How far will the cylinder roll up the incline?

$$\cancel{mgh_0} + \frac{1}{2}mv_0^2 + \frac{1}{2}I\omega_0 = mgh + \cancel{\frac{1}{2}mv^2} + \cancel{\frac{1}{2}I\omega^2}$$

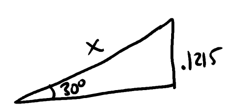
$$\frac{1}{2}mv_0^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v_0}{r}\right)^2 = mgh$$

$$\frac{1}{2}\cancel{m}v_0^2 + \frac{1}{4}\cancel{m}v_0^2 = \cancel{m}gh$$

$$\frac{1}{2}v_0^2 + \frac{1}{4}v_0^2 = gh$$

$$\frac{3}{4}v_0^2 = gh$$

$$\frac{3}{4}v_0^2/g = h = .1215m$$



$$x = \frac{.1215}{\sin 30°}$$

$$x = \underline{.243m}$$

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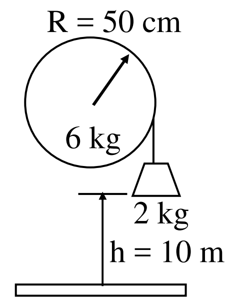
Conservation of Energy Problem 2

- Find the velocity of the mass just before it hits the ground.

$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I_D\omega^2$$
$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{1}{2}M_D R^2\right)\left(\tfrac{v}{r}\right)^2$$
$$mgh = \tfrac{1}{2}mv^2 + \tfrac{1}{4}M_D v^2$$
$$mgh = \left(\tfrac{1}{2}m + \tfrac{1}{4}M_D\right)v^2$$
$$\frac{mgh}{\left(\tfrac{1}{2}m + \tfrac{1}{4}M_D\right)} = v^2$$
$$\boxed{v = 8.85\ m/s}$$



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