Rotational Inertia and Torque

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Rotational Inertia

- Inertia is the tendency of an object to resist change.
- In translational motion, the inertia tells us how “easy” or “hard” it is to get object moving.
  - Based on the mass of the object.
- In rotational motion, the rotational Inertia tells us how “easy” or “hard” it is to get object spinning.
  - What is that based on?

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Rotational Inertia

- Rotational Inertia (or "Moment of Inertia") depends on the mass if the spinning object and where that mass is located (RADIUS)
- $I = \Sigma mr^2$ (units kg m^2)

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Inertia Rods

- Two rods have equal mass and length. Which will be "easier" to spin?

A) Mass on ends

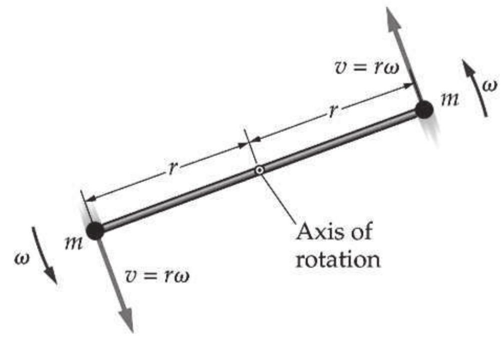
B) Same

C) Mass in center

$I = \Sigma m r^2$ Further mass is from axis of rotation, greater moment of inertia (harder to spin)

A Dumbbell

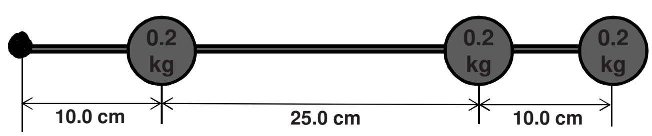
- Use the definition of moment of inertia to calculate that of a dumbbell-shaped object with two point masses m separated by a distance of $2r$ and rotating about a perpendicular axis through their center of symmetry.



$$I = \sum m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 = 2mr^2$$

Calculating Moment of Inertia

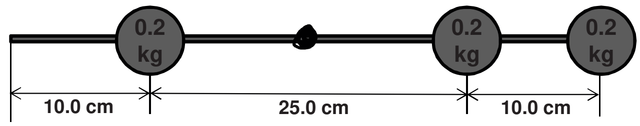
- What is the moment of inertia of the object below when rotated around the left end of the rod?



$$I = \Sigma mr^2 = .2(.1)^2 + .2(.35)^2 + .2(.45)^2 = .067 \ \text{kg}\cdot\text{m}^2$$

Calculating Moment of Inertia

- What is the moment of inertia of the object below when rotated around the middle of the rod?



$I = \Sigma m r^2 = (.2)(.125)^2 + (.2)(.125)^2 + (.2)(.225)^2$

$\approx .0164 \text{ kg} \cdot \text{m}^2$

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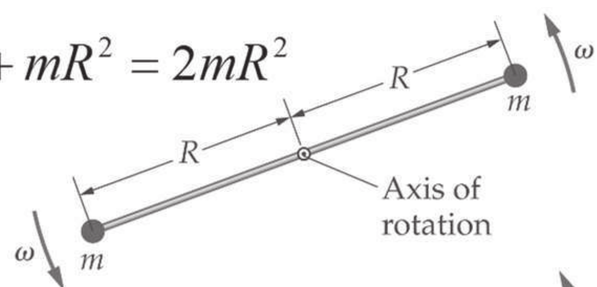
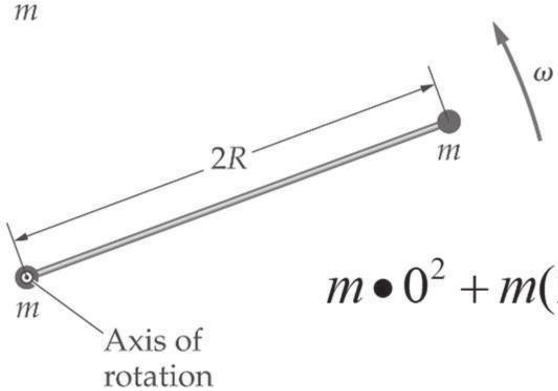
Rotational Inertia is Axis Dependent

$$mR^2 + mR^2 = 2mR^2$$





$$m \bullet 0^2 + m(2R)^2 = 4mR^2$$

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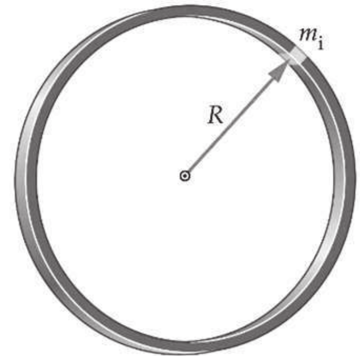
Moment of Inertia of a Hoop

$$\boxed{I = \sum m_i r_i{}^2}$$

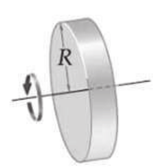


$$I = \sum m_i r_i^2 = \sum m_i R^2 = \left(\sum m_i\right) R^2 = MR^2$$

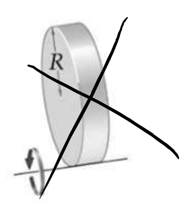
All of the mass of a hoop is at the same distance *R* from the center of rotation, so its moment of inertia is the same as that of a point mass rotated at the same distance.

Moments of Inertia

Hoop or
cylindrical shell
$I = MR^2$

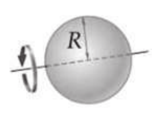


Disk or
solid cylinder
$I = \frac{1}{2} MR^2$

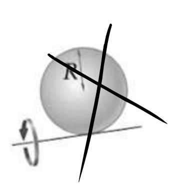


Disk or
solid cylinder
(axis at rim)
$I = \frac{3}{2} MR^2$

Hollow sphere
$I = \frac{2}{3} MR^2$



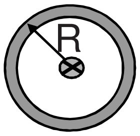
Solid sphere
$I = \frac{2}{5} MR^2$



Solid sphere
(axis at rim)
$I = \frac{7}{5} MR^2$

Calculating Moment of Inertia

- A circular hoop and a disk each have a mass of 3 kg and a radius of 20 cm. Compare their rotational inertias.

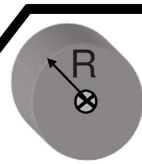


$$I = mR^2 = (3kg)(0.2m)^2$$

I = 0.120 kg m^2

$I = mR^2$

Hoop



$$I = \frac{1}{2}mR^2 = \frac{1}{2}(3kg)(0.2m)^2$$

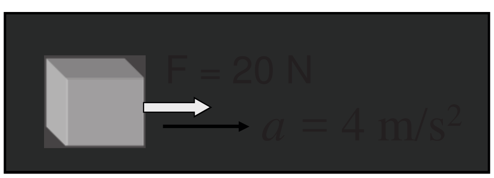
$I = ½mR^2$

Disk

I = 0.0600 kg m^2

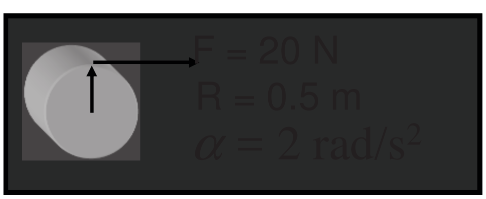
Torque and Rotational Inertia

- Newton's second law can be used to compare linear inertia (mass) and rotational inertia.



Linear Inertia, m

$$m = \frac{24\text{ N}}{4\text{ m/s}^2} = 5\text{ kg}$$



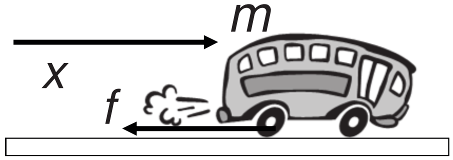
Rotational Inertia, $\boldsymbol{I}$

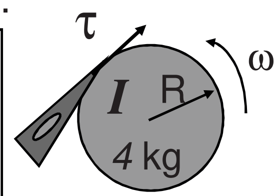
$$\boldsymbol{I} = \frac{\tau}{\alpha} = \frac{(20\text{ N})(0.5\text{ m})}{2\text{ rad/s}^2} = 5.0\text{ kg m}^2$$

Force does for translation what torque does for rotation.

Important Analogies

For many problems involving rotation, there is an analogy to be drawn from linear motion.





$\omega_o = 50$ rad/s

$\tau = 40$ N m

A resultant force *F* produces negative acceleration a for a mass *m*.

$$\boxed{F = ma}$$

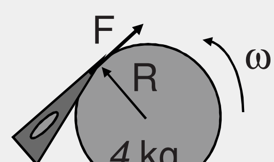
A resultant torque τ produces angular acceleration α of disk with rotational inertia *I*.

$$\boxed{\tau = I\alpha}$$

Newton's 2nd Law for Rotation

How many revolutions required to stop?

$$\boxed{\tau = I\alpha}$$



$\omega_o = 50$ rad/s

R = 0.20 m

F = 40 N

$$\cancel{F}\cancel{R} = (\tfrac{1}{2}m R\cancel{R})\alpha$$

$$\alpha = \frac{2F}{mr} = \frac{2(40N)}{(4kg)(0.2)}$$

$$\boxed{\alpha = 100 \text{ rad/s}^2}$$

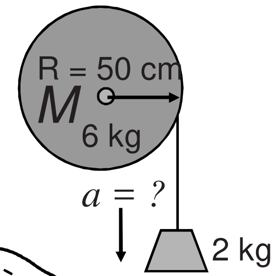
$$\overset{0}{\cancel{\omega^2}} = \omega_o^2 + 2\alpha\theta$$

$$\theta = \frac{-\omega_0^2}{2\alpha} = \frac{-(50 \text{ rad/s})^2}{2(100 \text{ rad/s}^2)}$$

$$\boxed{\theta = 12.5 \text{ rad} = 1.99 \text{ rev}}$$

Example

What is the linear acceleration of the falling 2-kg mass?



Apply Newton's 2nd law to rotating disk:

$\tau = I\alpha \Rightarrow T\cancel{R} = (\tfrac{1}{2}MR^{\cancel{2}})\alpha$

$T = \tfrac{1}{2}MR\alpha$ but $a = \alpha R; \quad \alpha = \frac{a}{R}$

$T = \tfrac{1}{2}M\cancel{R}(\frac{a}{\cancel{R}})$; and $T = \tfrac{1}{2}Ma$

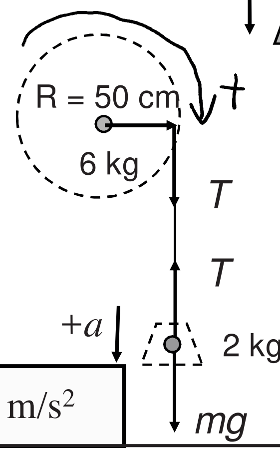
Apply Newton's 2nd law to falling mass:

$mg - T = ma \qquad mg - \tfrac{1}{2}Ma = ma$

$(2\text{ kg})(9.8\text{ m/s}^2) - \tfrac{1}{2}(6\text{ kg})\,a = (2\text{ kg})\,a$

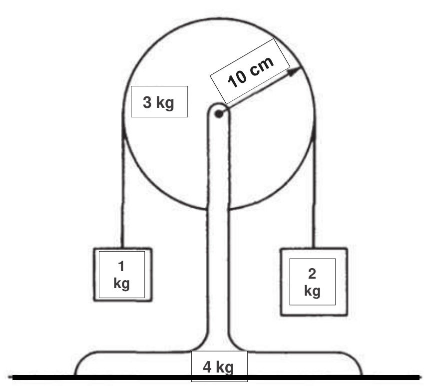
$19.6\text{ N} - (3\text{ kg})\,a = (2\text{ kg})\,a$

$\boxed{a = 3.92\text{ m/s}^2}$



Atwoods Revisited

- A pulley of mass 3 kg and radius 10 cm is mounted on frictionless bearings and supported by a stand of mass kg at rest on a table as shown above. The moment of inertia of this pulley about its axis is $0.5mr^2$.
- Passing over the pulley is a massless cord supporting a block of mass 1 kg on the left and a block of mass 2 kg on the right. The cord does not slip on the pulley, so after the block-pulley system is released from rest, the pulley begins to rotate.
- What is the acceleration of the two masses?
- What is the tension in the cord attached to the 2kg mass?



Atwoods Revisited(cont.)

$F_{NET_1} = M_1 A$ | $F_{NET_2} = M_2 A$

$T_1 - M_1 g = M_1 A$ | $M_2 g - T_2 = M_2 A$

$\tau_{NET} = I\alpha$

$\tau_2 - \tau_1 = I\alpha$

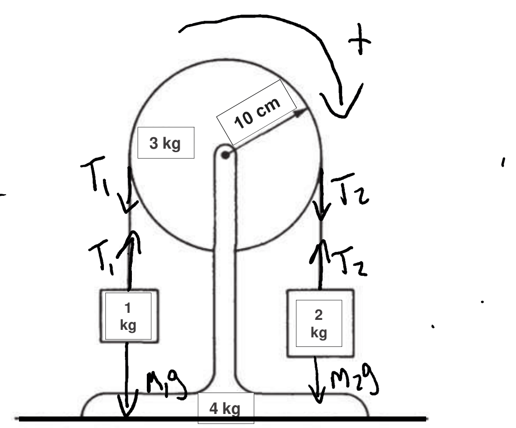
$T_2 \cancel{r} - T_1 \cancel{r} = \left(\frac{1}{2}(M) r^{\cancel{2}}\right) \frac{A}{\cancel{r}}$

$T_2 - T_1 = \frac{1}{2} M A$

$(M_2 g - M_2 A) - (M_1 A + M_1 g) = \frac{1}{2} M A$

$2(9.8) - 2A - A - 9.8 = \frac{1}{2}(3) A$

$-3A + 9.8 = 1.5A$



$T_1 = MA + M_1 g$

$4.5A = 9.8$

$A = 2.18 \, m/s^2$

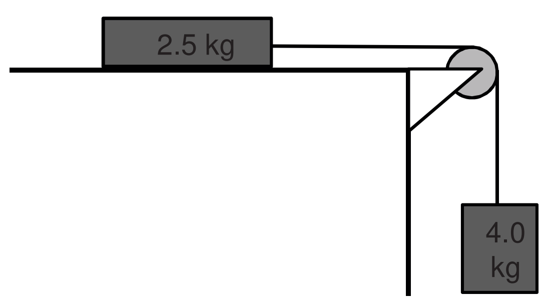
$T_2 = M_2 g - M_2 A$

$T_2 = 2(9.8) - 2(2.18)$

$\approx 15.24 \, N$

Atwoods 2

- The pulley has a mass of 3.0 kg and a radius of 2.0 cm.
- The pulleys mass is concentrated on it perimeter.
- Assume the surface is frictionless.
- Find the acceleration of the 4.0 kg mass shown.
- Find the tensions in the string.



Atwoods 2

$F_{NET_2} = T_2$

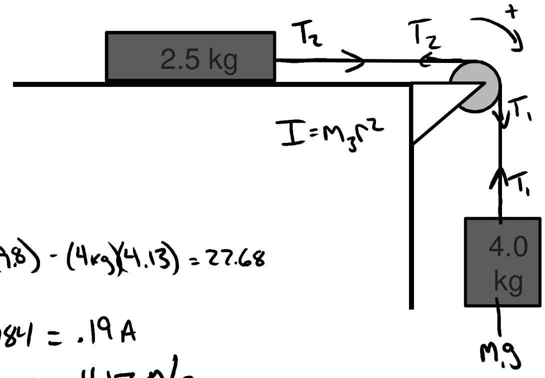
$M_2 A = T_2$

$F_{NET_1} = M_1 g - T_1$

$M_1 A = M_1 g - T_1$

$T_1 = M_1 g - MA$

$T_1 = (4kg)(9.8) - (4kg)(4.13) = 22.68$



$\tau_1 - \tau_2 = I\alpha$

$T_1 r - T_2 r = (M_3 r^2)\left(\frac{A}{r}\right)$

$(Mg - MA)r - (M_2 A)r = M_3 r A$

$[(4)(9.8) - 4(A)](.02) - (2.5A)(.02) = 3(.02)A$

$.784 - .08A - .05A = .06A$

$.784 = .19A$

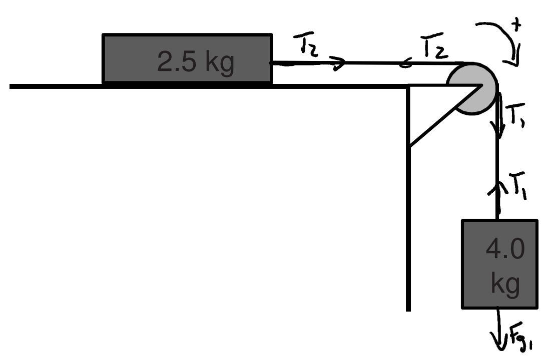
$A = 4.13\ m/s^2$

$T_1 = (4kg)(9.8) - (4kg)(4.13) = \underline{22.68N}$

$T_2 = (2.5)(4.13) = 10.33\ N$

Atwoods 3

- If the pulley from the last problem had a constant friction torque of 0.063 N•m, how would that change the acceleration of the mass?



$\tau_1 - \tau_2 - .063 = I\alpha$

$T_1 r - T_2 r - .063 = (M_D r^2)\frac{A}{r}$

$(Mg - M_1 A)r - (M_2 A)r - .063 = M_D r A$

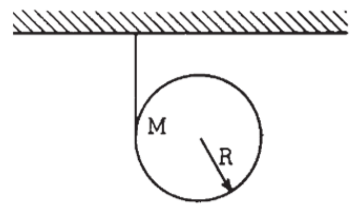
$Mgr - M_1 A r - M_2 A r - .063 = M_D r A$

$4(9.8)(.02) - 4(A)(.02) - (2.5)A(.02) - .063 = 3(.02)A$

$.784 - .08A - .05A - .063 = .06A$

$.721 = .19A$

$A = 3.79\ m/s^2$



- A cloth tape is wound around the outside of a uniform solid cylinder (mass M, radius R) and fastened to the ceiling as shown in the diagram above. The cylinder is held with the tape vertical and then released from rest. As the cylinder descends, it unwinds from the tape without slipping. The moment of inertia of a uniform solid cylinder about its center is ½MR^2.
- Label the forces acting on the cylinder.
- In terms of g, find the downward acceleration of the center of the cylinder as it unrolls from the tape.

Another Example

$F_{NET} = T_1 - Mg$

$-MA = T_1 - Mg$

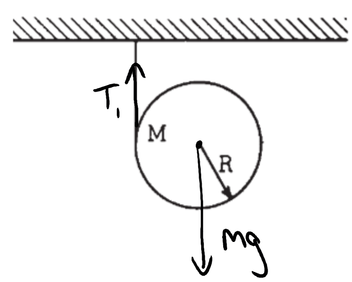
$T_1 = Mg - MA$

$\tau_{NET} = I\alpha$

$T_1 R = \frac{1}{2} MR^2 \left(\frac{A}{R}\right)$

$T_1 R = \frac{1}{2} MR\, A$

$T_1 = \frac{1}{2} MA$



$\frac{1}{2} MA = Mg - MA$

$\frac{3}{2} \cancel{M}A = \cancel{M}g$

$A = \frac{2}{3} g$