Projectile Motion

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Projectile Motion

- **Projectiles**
  - > **objects given an initial velocity that then move under the force of gravity**
- **Trajectory**
  - > **the path followed by a projectile**
  - > **The path is a curve called a parabola**

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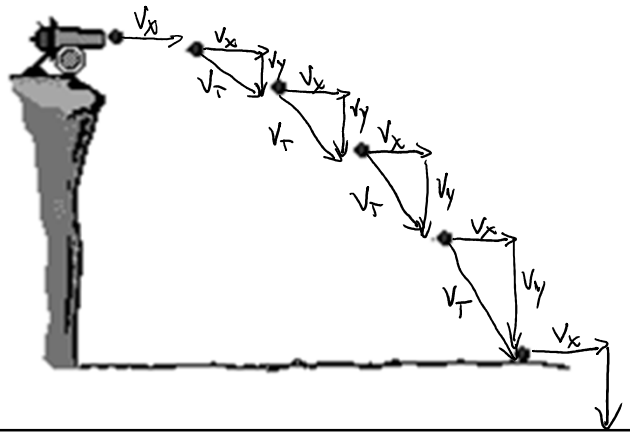
Velocity Vectors

- **How do the velocities in each direction change over time?**



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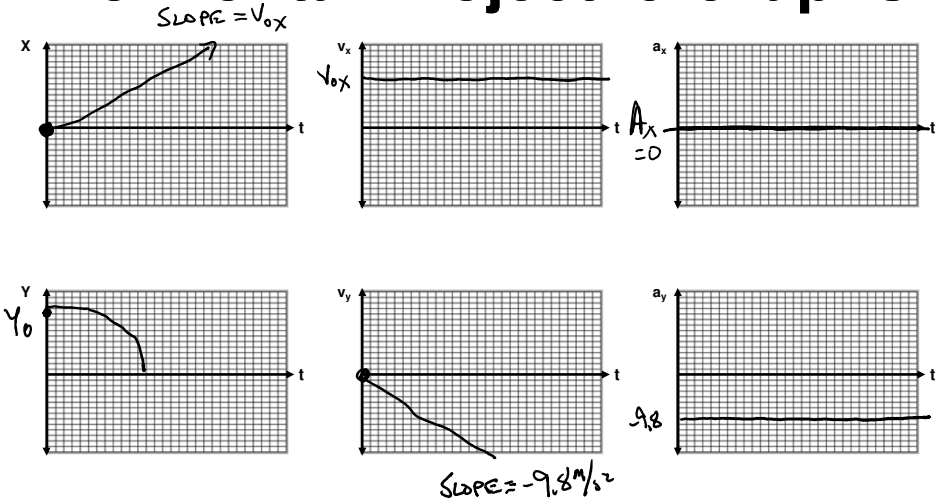
Horizontal Projectile Graphs



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Independence of Dimensions

- **Since the horizontal and vertical motion of an object are independent of each other, the motion equations can be used to determine the exact position of a projectile.**
  - **Time is the only the variable that links the two dimensions.**
- **However, we must first distinguish between the x and y components of any vectors.**

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Independence of Dimensions

- **With no acceleration in the horizontal direction, we can find the horizontal position by using the equation:**
  - $X = X_0 + V_{0x}t + \frac{1}{2}A_x t^2$ / $X = V_{0x}t$
- **The velocity in the horizontal direction will not change, therefore:**
  - $V = V_{0x} + A_x t$

    $V = V_{0x}$

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Independence of Dimensions

- **Since there is acceleration (gravity) in the vertical direction the position can be found using the equation:**
  >
- **The acceleration causes a change in velocity in the vertical direction. We can find the final velocity using the equations:**
  >
  >

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2$$

$$v_y = v_{0y} + a_y t$$

$$v_y^2 = v_{0y}^2 + 2a(y - y_0)$$

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Horizontal Projectile Problem

- **A stone is thrown horizontally at a speed of 15 m/s from the top of a cliff 78.4 m high.**
  - **How long is the stone in the air?**
  - **How far from the cliff does the stone land?**
  - **What is the horizontal and vertical components of the velocity just before the stone hits the ground?**

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Horizontal Projectile Problem

- **Find the time**

	X	Y
X	60	0
X_0	0	78.4
V	15	
V_0	15	0
A	0	$-9.8\ m/s^2$
t	4	4

SAME

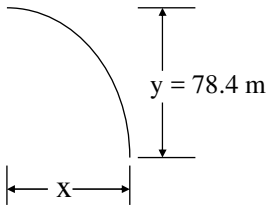
$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$

$0 = 78.4 + 0t^0 + \frac{1}{2}(-9.8)t^2$

$-78.4 = -4.9t^2$

$16 = t^2$

$4 = t$



Remember time is the same for both vertical and horizontal motion

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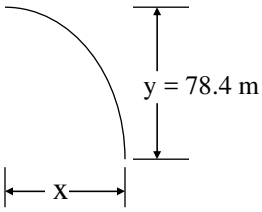
Horizontal Projectile Problem

- **Find the horizontal distance**

$X = v_0 t$

$= 15(4) = 60m$



- **Find the components of the final velocity** $V_x = 15$

$V_y = V_{0y} + A_y t = 0 + -9.8(4) = -39.2\ m/s$

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Horizontal Projectile Problem #2

- **You are at the top of a 3.5 m high stair case. A friend at the bottom of the stairs forgot her pencil and asks you for one. You notice that she is 8.5 m horizontally away from you. What horizontal velocity should you throw the pencil at to ensure that she gets the pencil?**

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Projectiles Launched at an Angle

- **When projectiles are launched at an angle, they are given an initial horizontal and vertical velocity.**
- **The horizontal distance the projectile travels is called the <u>range</u>.**

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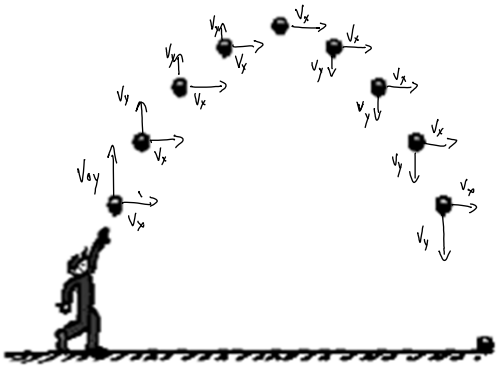
Velocity Vectors

- **How do the velocities in each direction change over time?**



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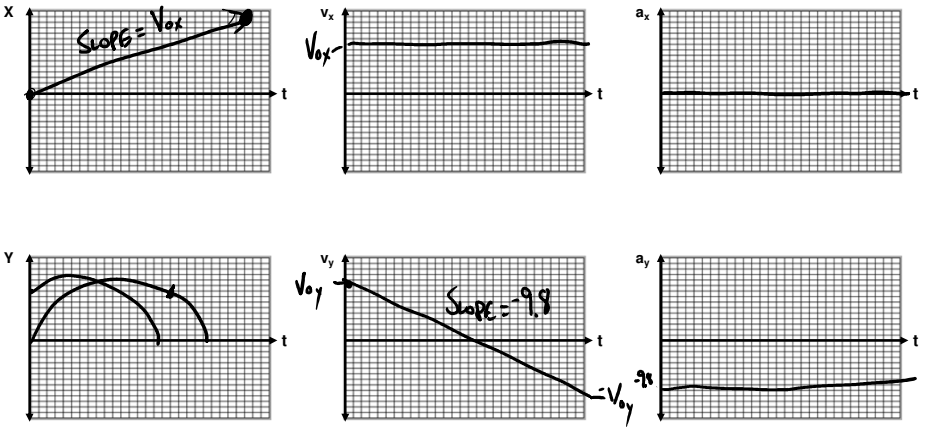
Projectile at an Angle Graphs



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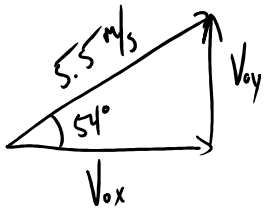
Angled Launch Problem

- **A ball is thrown with a initial velocity of 5.5 m/s at an angle of 54°. Find:**
  - > **the time in the air.**
  - > **how high the ball went.**
  - > **the range when it reaches it's launch height.**



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Angled Launch Problem

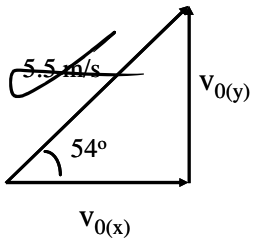
- **Find components**

$V_{ox} = 5.5 \cos 54°$

$= 3.23 \text{ m/s}$

$V_{oy} = 5.5 \sin 54°$

$= 4.45 \text{ m/s}$



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Angled Launch Problem

- **Find time**

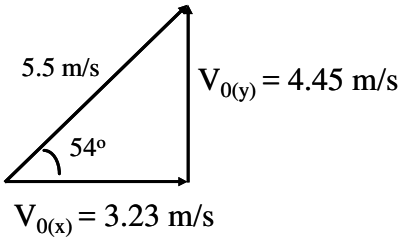
	X	Y
X_0, Y_0	0	0
X, Y		0
V_0	3.23	4.45
V	3.23	-4.45
A	0	-9.8
t		

$V_y = V_{0y} + At$

$-4.45 = 4.45 + -9.8t$

$-8.9 = -9.8t$

$t = .91s$



- Remember time is the same for both vertical and horizontal motion
- If the final vertical displacement is not zero, then use the quadratic formula

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Angled Launch Problem

- **Find Max height**

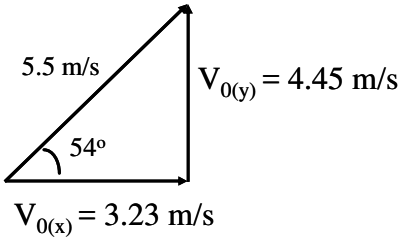
At Y_{MAX}, $V_y = 0$

$V_y^2 = V_{0y}^2 + 2A(Y_{MAX} - Y_0)$

$0^2 = 4.45^2 + 2(-9.8)(Y_{max} - 0)$

$-19.8 = -19.6\, Y_{max}$

$Y_{max} = 1.01m$



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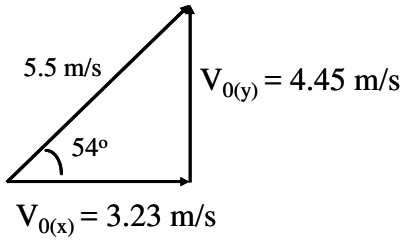
Angled Launch Problem

- **Find range**

$X = X_0 + V_{0x}t + \frac{1}{2}A_x t^2$

$= 0 + 3.23(.91) + \frac{1}{2}(0)(.91)^2$

$X = 2.94\text{m}$



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Projectiles Launched at an Angle

- **It can be proven using trigonometric identities that the range of the projectile can be found using:**

>

$$R = \frac{v_0^2}{g}\sin 2\theta$$

Note: This can only be used when the projectile is launched and lands at the same height. ($\Delta y = 0$)

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Angled Launch Problem #2

- **A ball is thrown with a initial velocity of 4.6 m/s at an angle of 33° from a office window 14.2 meters high. Find:**
  - > **the time in the air.**
  - > **how high the ball went.**
  - > **the distance away from the building where the ball will hit.**

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Angled Launch Problem #2

$V_{0x} = 4.6 \cos 33° = 3.86$ m/s

$V_{0y} = 4.6 \sin 33° = 2.51$ m/s

	X	Y
X_0, Y_0	0	14.2
X, Y		0
V_0	3.86	2.51
V	3.86	
A	0	-9.8
t		

At Y_{MAX}, $V_y = 0$

$V_y^2 = V_{0y}^2 + 2A(Y_{MAX} - Y_0)$

$0^2 = 2.51^2 + 2(-9.8)(Y_{MAX} - 14.2)$

$0 = 6.3 + -19.6\, Y_{MAX} + 278.32$

$-284.32 = -19.6\, Y_{MAX}$

$Y_{MAX} = 14.52$ m

Find V_y

$V_y^2 = V_{0y}^2 + 2A(Y - Y_0)$

$V_y^2 = 2.51^2 + 2(-9.8)(0 - 14.2)$

$V_y^2 = 284.6$

$V_y = \pm 16.87$ m/s $\Rightarrow$ -16.87 m/s at bottom

Find Time

$V = V_0 + At$

$-16.87 = 2.51 + (-9.8)t$

$t = 1.98$ s

Find Range

$X = V_{0x} t$

$= 3.86(1.98)$

$= 7.64$ m

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Angled Launch Problem #2

- **What is the total velocity at the max height, launch height, and just before it hits the ground?**

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