Making Things Rotate

How do you make an object rotate?

- Apply a force at point on the object other than its center of gravity.
- This quantity is defined as the torque on an object.

$$\tau = rF(\sin\theta)$$

Where θ is the angle between the lever arm and the applied force, and r is the distance between the applied force and the pivot point.

- Clockwise torque = Negative
- Counter-Clockwise torque = Positive

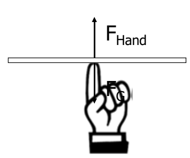
Equilibrium

The stick is in equilibrium:

- $\Sigma F_X = 0N$
- $\Sigma F_Y = 0N$
- $\Sigma \tau = 0$ Nm

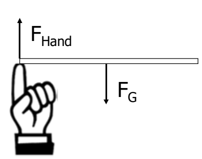
Translational Equilibrium (ΣF_X, ΣF_Y)

Rotational Equilibrium ($\Sigma \tau$)



The second stick is not:

- $\Sigma F_X = 0N$
- $\Sigma F_Y = 0N$
- $\Sigma \tau \neq 0$ Nm

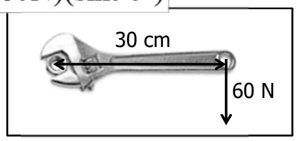


Example

- Suppose a 30 centimeter long steel wrench is used to tighten a bolt, with an force of 60 Newtons applied to the far end perpendicular to the wrench handle. How much torque is generated?

$$\tau = rF(\sin\theta) = (0.3\text{m})(60\text{N})(\sin 90°)$$

$$\tau = -18\,\text{N}\cdot\text{m}$$



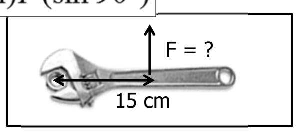
Example

- Suppose another person tries to loosen the bolt by applying a force midway down the handle. What force would have to be applied to <u>loosen</u> the bolt?

$$\tau = rF(\sin\theta)$$

$$18\,\text{N}\cdot\text{m} = (0.15\text{m})F(\sin 90°)$$

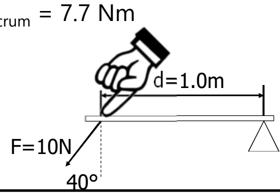
$$120\,\text{N} = F$$



Torque

Forces don't always act at 90° lever arm.

- Find the torque about the fulcrum:
  - $\tau_{\text{Fulcrum}} = \tau$
  - $\tau_{\text{Fulcrum}} = r\,F(\sin\theta)$
  - $\tau_{\text{Fulcrum}} = (1.0\text{m})(10\text{N})(\sin 50°)$
  - $\tau_{\text{Fulcrum}} = 7.7$ Nm

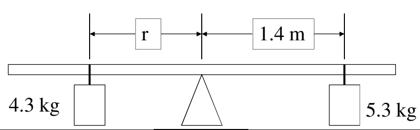


Rotational Equilibrium

- Like translational equilibrium and net force, rotational equilibrium is the non-rotating state of an object when the net torque is zero.

Example

- Where would a 4.3 kg bucket need to be placed to place this system in equilibrium?

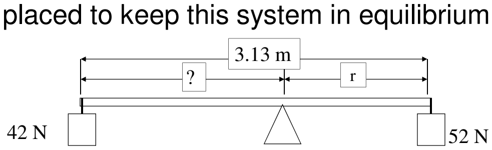


$$\tau_{net} = 0$$
$$Fr = Fr$$
$$(4.3kg)(9.8\frac{m}{s^2})r = (5.3kg)(9.8\frac{m}{s^2})(1.4m) \Rightarrow \boxed{r = 1.7m}$$

- What is the force on the fulcrum?

Example

- Where should the pivot point be placed placed to keep this system in equilibrium?



$42N(3.13 - r) = 52r$

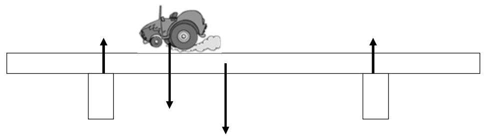
$131.46 - 42r = 52r$

$131.46 = 94r$

$r = 1.4m$ From 52N

Torque on Bridges

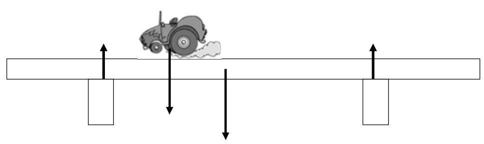
or any other structure with two supports



- What forces are acting in the vertical direction?
  - Car's weight
  - Bridge weight
  - Column support

Torque on Bridges

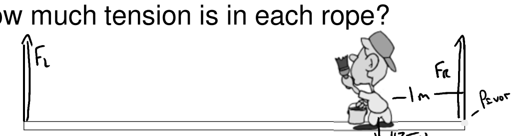
or any other structure with two supports



- The bridge must be in both translational and rotational equilibrium. Why?
  - It's not moving

Example

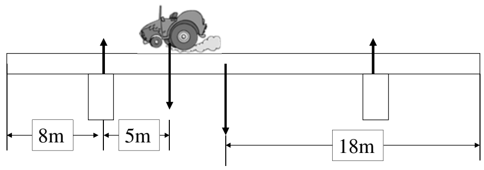
- A light weight scaffold is supported by a rope at each end and a painter is standing 1.0 m from the right end. If the painter weighs 435 N and the scaffold is 5 m long, how much tension is in each rope?



$\tau = 0 \Rightarrow F_R(0) - 435(1) + F_L(5)$ $\quad F_R + F_L = 435$

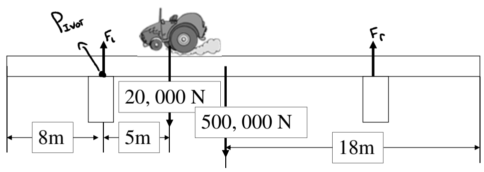
$435 = F_L(5) \Rightarrow F_L = 87$ $\quad F_R = 435 - 87 = 348\text{N}$

Torque on Bridges



- Let's say that the car has a weight of 20,000 N and the bridge weighs 500,000 N, how is the weight distributed on the two columns?

Torque on Bridges



$\tau = 0 \Rightarrow F_L(0) + (20{,}000)(5) + (500{,}000)(10) - F_r(20)$

$0 = 0 + 100{,}000 + 5{,}000{,}000 - F_r(20)$

$-5{,}100{,}000 = -F_r(20)$

$255{,}000\,N = F_r$

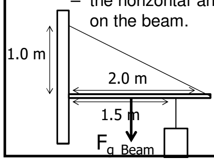
$F_R + F_L = 520{,}000$

$F_L = 520{,}000 - 255{,}000$

$F_L = 265{,}000\,N$

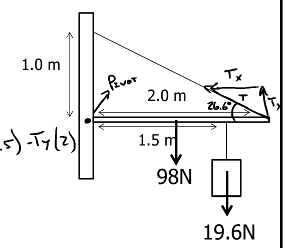
Example

- A horizontal beam is attached to the wall as shown. The beam has a mass of 10 kg. There is a weight hanging from the beam which has a mass of 2.0 kg. A rope is attached to the far end of the beam and connected to the wall a as shown.
- Solve for
  - the tension in the rope
  - the horizontal and vertical components of the force by the wall on the beam.



Example (Cont.)

- Choose Pivot Point
- Break Tension in x and y
- Solve for T_y



$\tau = 0 = F_w(0) + 98(1m) + 19.6(1.5) - T_y(2)$

$0 = 98 + 29.4 - 2T_y$

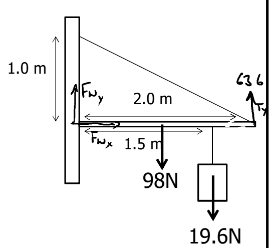
$-127.4 = -2T_y$

$T_y = 63.7\ N$

$\sin 26.6° = \frac{63.7\ N}{T} \quad T = 142.3\ N$

Example (Cont.)

- Use Translational Equilibrium to solve for the force the wall supplies to the beam.



$F_{w_y} + 63.6 = 98 + 19.6$

$F_{w_y} = 54\ N$

$F_{w_x} = -T_x$

$\tan 26.6° = \frac{63.6}{T_x}$

$T = 127\ N = F_{w_x}$