Gravity

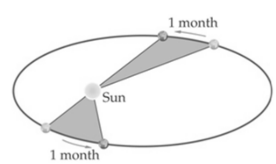
and

Planetary Motion

1

Kepler's Three Laws of Planetary Motion

- 1st Law
  - All Planets move in elliptical orbits with the sun at one focus
- 2nd Law
  - A line joining the planet to the sun sweeps out equal area in equal time. (Planets move faster when closer to the sun)



2

Kepler's Three Laws of Planetary Motion

- 3rd Law
  - The square of the period of any planet is proportional to the cube of the planet's mean distance from the sun

$$T^2 = \frac{4\pi^2}{GM} r^3$$

Can be used for any object revolving around another.

T = period of the satellite **in seconds**

G = Gravitational Constant (6.67 x 10^{-11})

M = Mass of the object that is being orbited

r = distance between the center of the planet and the center of the sun

3

Kepler's Three Laws of Planetary Motion

- 3rd Law
  - For any objects orbiting the same planet or star:

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$$

Earth's Period around the sun = 365.25 days

Average distance from the sun to the Earth = 1.5 x 10^{11} m or 1 AU

4

Kepler's Three Laws of Planetary Motion

- 3rd Law (Example)
  – If it takes 686.95 days for Mars to revolve around the sun, what is its mean distance from the sun?

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3 \Rightarrow \left(\frac{T_{Mars}}{T_{Earth}}\right)^2 = \left(\frac{r_{Mars}}{r_{Earth}}\right)^3$$

$$\Rightarrow \left(\frac{686.95\,days}{365.25\,days}\right)^2 = \left(\frac{r_{Mars}}{1.0\,AU}\right)^3$$

$$r_{Mars} = 1.52\,AU$$

5

Newton's Universal Law of Gravity

- Any two objects of mass, m_1 and m_2 are accelerated towards each other by a force due to gravity.

For Earth, = 9.8 m/s^2

$$F = G\frac{m_1 m_2}{r^2}$$

G = 6.67 x 10^{-11} N·m^2 / kg^2
M_E = 5.98 x 10^{24} kg
R_E = 6.38 x 10^6 m

6

Newton's Universal Law of Gravity

- For any object of mass, m, that is a certain distance from the surface of the Earth.

$$F = G\frac{M_E m}{(R_E + h)^2}$$

$G = 6.67 \times 10^{-11}\ N \cdot m^2 / kg^2$
$M_E = 5.98 \times 10^{24}$ kg
$R_E = 6.38 \times 10^6$ m

7

Gravitational Field

- The acceleration felt on a mass due to a gravitational force
- In general, the acceleration due to gravity is:

$$g(r) = \frac{GM_E}{r^2}$$

where r is the distance from the center of the earth

- For any distance above the earth, $r = R_E + h$

$$g(r) = \frac{GM_E}{(R_E + h)^2}$$

8

Gravitational Field

- How far above the surface of the earth must you be to have an acceleration due to gravity that is 85% of the gravity at the surface?

$$g(r) = \frac{GM_E}{(R_E + h)^2} \Rightarrow (.85)(9.8) = \frac{(6.67x10^{-11})(5.98x10^{24})}{(6.38x10^6 + h)^2}$$

$$\Rightarrow 8.33 = \frac{3.989x10^{14}}{(6.38x10^6 + h)^2} \quad \Rightarrow (6.38x10^6 + h)^2 = 4.789x10^{13}$$

$$\Rightarrow 6.38x10^6 + h = 6.920x10^6 \Rightarrow h = 5.40x10^5 \text{ m}$$

9