Circular Motion II

1

Circular Acceleration

- Centripetal Acceleration
- Centripetal => “center seeking”
  - $$a_c = \frac{v^2}{r}$$
- Substituting for v…
  - $$a_c = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$$

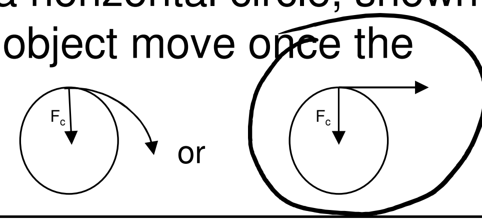
2

Circular Forces

- Centripetal Force
  - force applied to the object to keep it moving in a circle.

$$F_c = ma_c = \frac{mv^2}{r}$$
$$= mr\omega^2$$

- Consider an object moving in a horizontal circle, shown below. What direction will the object move once the centripetal force disappears?



3

Circular Forces: Application

- A car moves around a curve that has a radius of 45.0 m. If the concrete pavement is dry (μ = 0.8), what is the maximum speed that the car can move around the curve without skidding?

  - What is keeps the car from skidding off the track?
    - Friction

4

Circular Forces: Application

- So the force of friction must apply the centripetal force or:

$$F_f = F_c \Rightarrow \mu F_N = \frac{mv^2}{r} \Rightarrow \mu \cancel{m} g = \frac{\cancel{m} v^2}{r}$$

- Solving for velocity, we get:

$$v^2 = \mu g r \Rightarrow v = \sqrt{\mu g r} \Rightarrow v = \sqrt{.8(9.8)(45)}$$

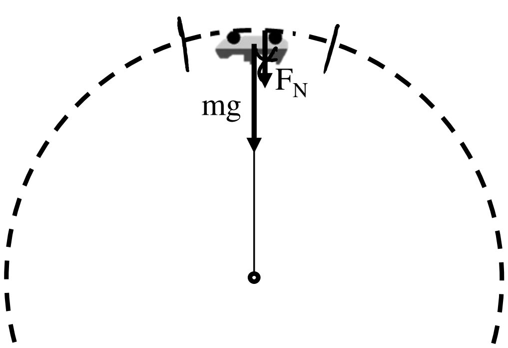
$$v = 18.8 \text{ m/s}$$

5

Vertical Circles

- Draw a free body diagram for a roller coaster car traveling in a vertical loop.

$$F_{net} = F_N + mg$$



Since both forces are pointing toward the center, we can call the net force the centripetal force

$$F_c = F_N + mg$$

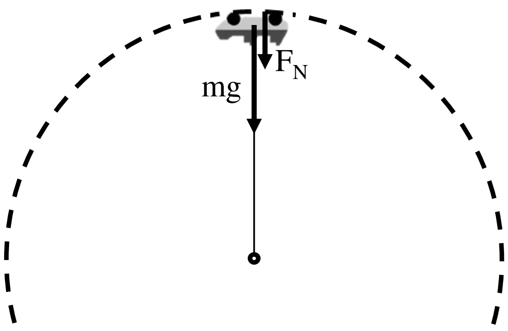
If the car travels just fast enough to make it around the loop, then the track will not need to provide any force, $F_N = 0$

$$F_c = mg$$

6

Vertical Circles

- Substituting and rearranging, we can find the minimum velocity that the car travel to just make it around the loop:



$$F_c = mg$$

$$\frac{mv^2}{r} = mg$$

$$v^2 = gr$$

$$v = \sqrt{gr}$$

7

Rotary Motion: Acceleration

- Angular Acceleration (α)
  – rate of change of angular velocity

$A = \frac{\Delta V}{\Delta t}$

$$\alpha = \frac{\Delta \omega}{\Delta t} \Rightarrow \frac{RAD/s}{s} \Rightarrow RAD/s^2$$

LINEAR	ANG
x	θ
V	ω
A	α

- Angular and Linear Acceleration
  – Like linear velocity, linear acceleration also varies with the distance from the center of motion, therefore:

$$a_t = r\alpha$$

8

Linear vs. Rotary

- All rotary equations follow the same format as their linear counterparts

Linear		Rotational
$x = x_o + v_o t + \frac{1}{2}at^2$	1	$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$
$v = v_o + at$	2	$\omega = \omega_o + \alpha t$
$v^2 = v_o^2 + 2a(x - x_o)$	3	$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$

9

Sample Problem

- The blades of a fan running at low speed turn at $\overset{\omega_o}{\cancel{275}}$ rpm. 28.8 RAD
  When the fan is switched to high speed, the rotation rate increases uniformly to $\overset{\omega}{\cancel{360}}$ rpm in 5.03 s. 37.7 RAD/s
  - What is the magnitude of the angular acceleration of the blades? α
  - How many revolutions do the blades go through while the fan is accelerating? $\theta \Rightarrow$ CONVERT

$$\frac{275 \text{ REV}}{1 \text{ MIN}} \cdot \frac{2\pi \text{ RAD}}{1 \text{ REV}} \cdot \frac{1 \text{ MIN}}{60 \text{ s}}$$

10

Sample Problem

- What is the magnitude of the angular acceleration of the blades? α?

$\omega_0 = 28.8$
$\omega = 37.7$
$t = 5.03s$

$\omega = \omega_0 + \alpha t$
$37.7 = 28.8 + \alpha(5.03)$
$8.9 = \alpha(5.03)$
$\alpha = 1.77 \text{ RAD}/s^2$

- How many revolutions do the blades go through while the fan is accelerating?

$\Theta = 0 + 28.8(5.03) + \frac{1}{2}(1.77)(5.03)^2$

$\Theta = 167.26 \text{ RAD}$

$\frac{167.26 \text{ RAD}}{} \Big| \frac{1 \text{ REV}}{(2\pi \text{ RAD})}$

$\Theta = \underline{26.6 \text{ REVS}}$

11